Sheet 2

P4)

a)

The Number-density of YAG formula units ~~is~~ calculated as follows:

Molar density: $\rho \cdot \frac{1}{M} = 4.55 \frac{g}{mol} \times \frac{1}{593.6} \frac{mol}{g}$

$\Rightarrow$ Number density is given by

$$\rho \cdot \frac{N_A}{M}$$

As each formula unit contains 3 Y^{3+} ions that can act as possible place for an active ion, the total number density of of dopant ions is given by

$$3 \cdot \rho \cdot \frac{N_A}{M} \cdot \frac{w[\%]}{100}$$

with $w[\%]$ = atomic percentage of doping ions with respect to Y^{3+} ions.

$$\Rightarrow \quad N = \frac{3}{100} \cdot \rho \cdot \frac{N_A}{M} = 1.385 \cdot 10^{+20} \, cm^{-3}$$

$$= 1.385 \cdot 10^{26} \, m^{-3}$$

P4) b)

$$I = I_0 \cdot e^{-\sigma_a \cdot N \cdot L}$$

$$\Rightarrow \quad T = \frac{I}{I_0} = e^{-\sigma_a N L}$$

$$\Leftrightarrow \quad -\sigma_a N L = \ln T$$

$$\sigma_a = -\frac{1}{NL} \cdot \ln T$$

$$= -\frac{1}{0.5 \cdot 1{,}38 \cdot 10^{26}\,m^{-3} \cdot 0.01\,m} \cdot \ln 0.205$$

$$= 2.3 \cdot 10^{-24}\,m^2$$

P5

a)

$$g(\nu)\,d\nu = \frac{2}{\pi} \frac{\Delta\nu}{4(\nu-\nu_0)^2 + \Delta\nu^2}\,d\nu$$

$$= \frac{2}{\pi} \frac{\Delta\lambda \cdot \frac{c}{\lambda_0^2}}{4\left(\frac{c}{\lambda} - \frac{c}{\lambda_0}\right)^2 + \frac{\Delta\lambda^2}{\lambda_0^4} c^2} \frac{c}{\lambda^2}\,d\lambda$$

$$= \frac{2}{\pi} \frac{\Delta\lambda}{4c^2\left(\frac{\lambda_0 - \lambda}{\lambda\lambda_0}\right)^2 + \frac{\Delta\lambda^2}{\lambda_0^4} c^2} \frac{c^2}{\lambda_0^2\lambda^2}\,d\lambda$$

$$= \frac{2}{\pi} \frac{\Delta\lambda}{4(\lambda-\lambda_0)^2 + \Delta\lambda^2 \cdot \frac{\lambda^2}{\lambda_0^2}}\,d\lambda$$

due to $\Delta\lambda \ll \lambda_0 \Rightarrow \lambda \approx \lambda_0$

$$\Rightarrow g(\lambda)\,d\lambda = \frac{2}{\pi} \frac{\Delta\lambda}{4(\lambda-\lambda_0)^2 + \Delta\lambda^2}$$ qed.

P5)

b)

Using the Füchtbauer-Ladenburg Equation, we find

$$\frac{1}{\tau} = 8\pi n^2 c \int \frac{\sigma_e(\lambda)}{\lambda^4} d\lambda$$

$$\approx 8\pi n^2 c \int \frac{\sigma_e(\lambda)}{\lambda_0^4} d\lambda \qquad \text{for } \Delta\lambda << \lambda_0$$

$$= \frac{8\pi n^2 c}{\lambda_0^4} \int \sigma_e(\lambda) d\lambda$$

$$= \frac{8\pi n^2 c}{\lambda_0^4} \cdot \int \sigma_e(\lambda_0) \Delta\lambda \frac{\pi}{2} \cdot g(\lambda) d\lambda$$

$$= \frac{4\pi^2 n^2 c}{\lambda_0^4} \Delta\lambda \, \sigma_e(\lambda_0) \underbrace{\int g(\lambda) d\lambda}_{=1}$$

$$\Rightarrow \boxed{\sigma_e(\lambda_0) \cdot \tau = \frac{\lambda_0^4}{4\pi^2 n^2 c \Delta\lambda}}$$

P5c

$$\Delta\nu = \frac{1}{2\pi\tau} \quad \Rightarrow \quad \Delta\lambda = \frac{\lambda_0^2}{c}\Delta\nu = \frac{\lambda_0^2}{2\pi c\tau}$$

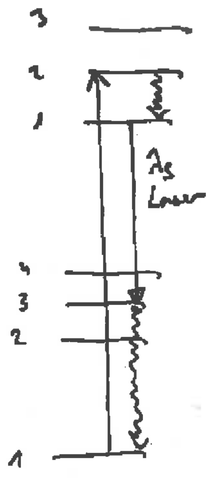
$$\Rightarrow \quad \sigma_e(\lambda_0) = \frac{\lambda_0^4}{4\pi^2 c\tau\Delta\lambda} = \frac{\lambda_0^4}{4\pi^2 \cancel{c\tau}\cdot\lambda_0^2}\, 2\pi\cancel{c\tau}$$

$$\Rightarrow \quad \sigma_e(\lambda_0) = \frac{\lambda_0^2}{2\pi} \qquad \text{qed.}$$

P6/ a)

$$\lambda_s = \frac{1}{E_{21} - E_{13}} = 1.0293\,\mu m$$

$$\lambda_p = \frac{1}{E_{22} - E_{11}} = 941{,}3\,\mu m$$



$$Z_1 = \sum_{i \in 1} d_{1i}\, e^{-\frac{E_{1i}}{k_B T}}$$

$$= 2.28571$$

$$Z_2 = \sum_{i \in 2} d_{2i}\, e^{-\frac{E_{2i}}{k_B T}}$$

$$= 8.8228 \cdot 10^{-22}$$

$$\Rightarrow \quad \text{Pump}: \quad f_{1,1} = \frac{1}{Z_1} \cdot d_{11} \cdot e^{-\frac{E_{11}}{k_B T}} = \frac{d_{10}}{Z} = 0.875$$

$$\text{Laser}: \quad f_{2,1} = \frac{1}{Z_2}\, d_{21} \cdot e^{-\frac{E_{21}}{k_B T}} = 0.702$$

P6

b)

$$\lambda_p = \frac{hc}{k_B T}\left[\ln \frac{Z_1}{Z_2}\right]^{-1} = \underline{\underline{972{,}7\ nm}}$$

c)

$$\frac{\sigma_e}{\sigma_a} = e^{-\frac{hc}{k_B T}\left(\frac{1}{\lambda} - \frac{1}{\lambda_p}\right)}$$

$$T = 300\,K \Rightarrow \frac{hc}{k_B T} = \underline{47{,}959\ \mu m}$$

reabsorption : $\sigma_a(\lambda_s) = \sigma_e(\lambda_s)\cdot e^{\frac{hc}{k_B T}\left(\frac{1}{\lambda_s} - \frac{1}{\lambda_p}\right)}$

$$= 2.31\cdot 10^{-20}\,cm^2 \cdot e^{47.959\mu m\cdot\left(\frac{1}{1029\mu m} - \frac{1}{0.9727\mu m}\right)}$$

$$\underline{\underline{= 1.56\cdot 10^{-21}\,cm^2}}$$

backemission : $\sigma_e(\lambda_p) = \sigma_a(\lambda_p)\cdot e^{-\frac{hc}{k_B T}\left(\frac{1}{\lambda_p} - \frac{1}{\lambda_p}\right)}$

$$= 7.6\cdot 10^{-21}\,cm^2 \cdot e^{-47.959\mu m\left(\frac{1}{0{,}941\mu m} - \frac{1}{0.9727\mu m}\right)}$$

$$= \underline{\underline{1{,}44\cdot 10^{-21}\,cm^2}}$$